Singular Propositions
and Modal Logic

Christopher Menzel
Texas A&M University

THE RUSSELLIAN PUZZLE

According to the prevailing view in the philosophy of language nothing mediates semantically between a proper name (in a speaker’s mouth) and its reference. Rather, the reference of a name is determined directly; there is no more to the semantics of a name than the object referred to. A thesis that often accompanies this widely shared view is that the meaning of a sentence \( \varphi \) is an abstract entity, the proposition \([\varphi]\) it expresses. If \( \varphi \) contains a name, the proposition expressed is said to be singular. This thesis in turn is often supplemented with the metaphysical thesis that propositions, singular propositions in particular, are structurally complex; that is, roughly, (i) that they have, in some sense, an internal structure that corresponds rather directly to the syntactic structure of the sentences that express them, and (ii) that the metaphysical components, or constituents, of that structure are the semantic values—the meanings—of the corresponding syntactic components of those sentences.¹ Let us refer to these three related theses jointly, for better or ill, as Russellian semantics.

Russellian semantics has an important consequence. Consider a sentence containing a name—the sentence ‘Quine is a philosopher’, say. If Russellian semantics is correct, it follows that the singular proposition this sentence expresses—the proposition \([\text{Quine is a philosopher}]\)—contains
Quine himself as a constituent in that part of its metaphysical structure corresponding to the name 'Quine'. Now, although in the physical realm wholes do not generally seem to presuppose their constituent parts (it's still the same car despite the new set of wipers), the same does not seem to be true of propositions. Rather, the individual constituents of a Russelian proposition seem not at all incidental to the nature of the proposition, but essential to it. It is hard, for example, to see how the proposition [Quine is a philosopher], if Russelian, could possibly have existed, could possibly have been what it is, sans Quine. (What, for instance, would have distinguished it from the proposition [Russell is a philosopher] had neither existed?) Thus, Russelian semantics appears to entail that propositions are ontologically dependent upon their constituents; i.e., more exactly, that

**OD** If \( x \) is a constituent of a proposition \( y \), then, necessarily, \( y \) exists only if \( x \) does.

But **OD** raises a puzzle. Intuitively,

(1) Quine might not have existed;

that is, the proposition [Quine does not exist] could have been true. However, given **OD**, this proposition could *not* have been true. For, as Adams notes, "a proposition must be in order to be true." But by **OD** there would have been no such proposition as [Quine does not exist] if Quine hadn't existed, hence it wouldn't have been there to be true; rather, there would have been no information about Quine whatever. Since of course [Quine does not exist] also fails to be true when Quine does exist, it follows that, necessarily, whether he exists or not the proposition that he doesn't is not true. Thus, Quine's nonexistence is not really possible, and hence, contrary to our initial intuition, (1) isn't true after all.

Call this the Russelian puzzle. My goal in this paper is to develop several logical solutions to this puzzle that preserve both the structured view of propositions—including its apparent consequence **OD**—and the intuition that (1) is true. The first of these is suggested in the writings of Arthur Prior, and, due to its awkwardness, is the least attractive. A simple modification of the semantical basis of Prior's solution suggested by Robert Adams guides the second, much more desirable, solution. Finally, a generalization of Adams' suggestion leads to what I find to be the simplest, most attractive solution to the Russelian puzzle.

**FRAMEWORK FOR A SOLUTION**

The first order of business in approaching the Russelian puzzle is to provide a proper representational framework, one capable of expressing the propositions and making the distinctions that appear in the puzzle. The puzzle, of course, is modal, essentially involving both counterfactual conditions and the notions of possibility and necessity. Assuming, reasonably, that the counterfactuals in the puzzle are necessary truths if true at all, the argument can be cast into an equivalent, though somewhat more stilted form that replaces the counterfactual conditions with strict entailment. We therefore need on these grounds adopt no more than a standard propositional modal base. Although quantifiers are not explicitly involved in the puzzle, the argument does involve a distinguished existence predicate 'E!', and surely, to characterize its logic correctly, we will want to be able to clarify its connection with the existential quantifier. Hence, an accurate representation of the puzzle requires the full expressive capacities of a first-order modal language.

Of course we cannot stop there. For the heart of the puzzle concerns Russelian propositions. More specifically, the puzzle requires that we be able to talk about such propositions, and in particular, at least, that we be able sensibly to ascribe both truth and existence to them. Furthermore, since the puzzle has to do with the ontological dependency of singular propositions on their constituents, we shall have to have some way of indicating that a given entity is a constituent of a given proposition. The most natural way of meeting this last requirement is simply to show constituency as in the informal presentation of the puzzle simply by allowing the formation of proposition denoting expressions \( e \) out of the formulas of the language, formulas containing names in particular. One might then express ontological dependency by means of the axiom schema \( \square(\forall e)(Ee = E\tau) \), where \( \tau \) is a term that occurs (free) in \( e \).

Proposition denoting expressions were constructed in the informal exposition above by simply bracketing sentences, and something along those lines will do here. However, bracketing of sentences is not sufficiently general, for analogous Russelian puzzles arise for properties and relations as well. Just as there are singular propositions, there are also singular properties and relations that involve individual constituents; for example, the property [being distinct from Quine]. Thus, it appears that **OD** generalizes to

**OD** If \( x \) is a constituent of a property, relation, or proposition \( y \), then, necessarily, \( y \) exists only if \( x \) does.

But then we can argue much as above. For instance, intuitively,

(1*) If Quine hadn't existed \( i \) would nonetheless have been distinct from him;

that is, I would nonetheless have had the property [being distinct from Quine]. As with propositions, a property must be in order to be exemplified. But by **OD***, there would have been no such property as [being distinct from
Quine] if Quine hadn't existed, and hence it wouldn't have been there to be exemplified. Hence, if Quine hadn't existed, I wouldn't have had the property of being distinct from him, and hence it appears that \((1^*)\) isn't true after all, a conclusion no less discomfiting than the one in our original puzzle.

As with propositions in the original puzzle, in this version we ascribe existence to properties. And, analogous to our talk in the original of a proposition's being true, here we talk about a property's being exemplified. Thus, a complete solution to the entire class of Russellian puzzles will require of us the means for talking directly about properties, relations, and propositions (PRPs) generally. For this purpose we will formalize our informal use of the bracket notation above to construct a class of PRP-denoting terms, or intensional abstracts. Specifically, where \(\varphi\) is any first-order formula and \(v_1, \ldots, v_n\) any variables, the expression \([\lambda v_1, \ldots, v_n \varphi]\) is a term that, intuitively, denotes the \(n\)-place relation expressed by \(\varphi\). Thus, when \(q\) is Quine, \([\lambda x \neq q]\) is the property (i.e., 1-place relation) [being distinct from Quine]. When \(n = 0\), intensional abstracts denote propositions. Thus, where \(E'\) is existence, \([\lambda \neg E' q]\) (or, suppressing the \(\lambda\), \([\neg E' q]\)), is the proposition [Quine doesn't exist]. Its ontological dependence on Quine can then be expressed as \(\Box [E' [\lambda \neg E' q] \supset E' q]\), and the dependence of [being distinct from Quine] on Quine by \(\Box [E' [\lambda x \neq q] \supset E' q]\). Using this apparatus, then, we can express \(OD^*\) for PRPs \([\lambda \varphi]\) generally as \(\Box E' [\lambda \varphi] \supset E' \tau\), where \(\tau\) is any constant or free variable occurring in \([\lambda \varphi]\).\(^3\)

As noted, in addition to being able to assert their existence, we also need to be able to express that certain propositions are true, and, in the general case, that certain PRPs are exemplified. As with existence, we could just add truth and exemplification predicates to our language. But since we are only interested in exploring the logic of the Russellian puzzle, it would be far more preferable if we could avoid embroiling ourselves in the formidable logical complexities of these predicates.\(^4\)

Toward this end, note that PRPs play two metaphysical roles: an objectual role and a predicative role, a role they play in being truly predicated of other objects. In their objectual role, PRPs can be referred to and quantified over no less than any other kind of object. This role is reflected in our use of that-clauses, gerunds, infinitives, and other PRP-denoting expressions, and in the legitimacy of, e.g., existentially generalizing over such expressions, as in the inference from, say, 'John believes that Bach never composed for guitar' to 'John believes something'. In their predicative role they are predicated of other objects (or, in the case of propositions, they are, so to say, predicated of the world), a role reflected in ordinary English in our use of verb phrases and declarative sentences. In our logical language, the former role is of course reflected in our complex terms, and the latter in the usual way in the predicates of the language. Now, with possible exceptions brought to light by Russell's paradox (which we shall studiously ignore), the truth conditions for a predication are exactly those of the corresponding exemplification statement—if \(\forall\) is a predicate expressing some property \(p\), \([\lambda \varphi]\) a complex term denoting \(p\), \(\Delta\) a 2-place exemplification predicate, and \(a\) the object denoted by a term \(\tau\), then both the predication \([\lambda \varphi] a\) and the corresponding exemplification statement \(\Delta ([\lambda \varphi], \tau)\) are true just in case \(a\) is in the extension of \(p\). However, since the predication involves only the property or relation being predicated and the object(s) of the predication without the mediation of the exemplification relation, we can avoid a special exemplification predicate in expressing the general Russellian puzzle if only we have a sufficiently robust representation of complex PRPs in their predicative role.

However, as things stand, because there are only complex terms, our logical language has in general no direct way of expressing the predication of a given complex PRP of a given object; in particular, one cannot in any natural way predicate the property [being distinct from Quine] of me. What we need, of course, is a representation of the unnominalized verb phrase 'is distinct from Quine' corresponding to the complex term \([\lambda x \neq q]\) that represents the nominalization 'being distinct from Quine'. We could of course introduce a class of new complex predicates to meet this need, but that would be unnecessary. For if we accept that it is the same PRP indicated by a verb phrase and its nominalization,\(^5\) albeit indicated in different roles, then there is no reason not to let our complex PRP-denoting expressions serve as both terms and predicates, and thereby play the grammatical roles of both nominalized verb phrases and their unnominalized counterparts. Call this dual-role syntax. Extending the duties of complex terms in this fashion, we represent my exemplifying distinctness from Quine by simply predicating it of me thus: \([\lambda x \neq q] c\); and in this fashion, as indicated, an explicit exemplification predicate and its attendant difficulties are avoided.\(^6\)

What about the truth predicate? Truth is naturally thought of as a special case of exemplification. An \(n + 1\)-place exemplification predicate \(\Delta_{n+1}\) is true of its arguments \(\tau_1, \ldots, \tau_n\) just in case \(\tau\) denotes an \(n\)-place relation \(R\) that is true of the objects \(a_1, \ldots, a_n\) denoted by \(\tau_1, \ldots, \tau_n\). In the case where \(n = 0\), the 1-place exemplification predicate \(\Delta\) takes only a single term \(\tau\) as its argument, and is true of \(\tau\) just in case \(\tau\) denotes a 0-place relation \(P\)—i.e., a proposition—that is true simpliciter. By allowing \(n\)-place complex terms to serve also as predicates, we avoided the need for special exemplification predicates to represent the second form of the Russellian puzzle above; predication alone—indicated by concatenation—suffices: The \(n\)-place predication \([\lambda \varphi_1, \ldots, \varphi_n] \tau_1 \ldots \tau_n\) is true iff \(a_1, \ldots, a_n\) exemplify (stand in) the relation \(R\) denoted by \([\lambda \varphi_1, \ldots, \varphi_n]\). Truth is just a special case. In the limiting case where \(n = 0\), a 0-place term standing alone will suffice: The 0-place predication \([\lambda \varphi]\) is true iff the proposition \(P\) it denotes is true. Thus, assertions to the effect that a certain proposition \(P\) is true, as
in the original form of the puzzle, in our logical language simply take the form \([\lambda \varphi]\), where \([\lambda \varphi]\) expresses \(P\). We can thus have all the expressive power we need for examining the logic of the Russellian puzzle without emboiling ourselves in the formidable difficulties of truth and exemplification. For readability, however, we introduce an eliminable “pseudo” truth predicate ‘\(T\)’ such that \(T[\varphi] \equiv_{df} [\varphi]\), so that the puzzle in the garb of dual-role syntax corresponds more closely to the logical form of the puzzle as expressed in ordinary language.

THE LOGICAL STRUCTURE OF PRPs

As noted above, a sufficiently robust notion of a singular property or proposition requires that PRPs exhibit some sort of structural complexity corresponding more or less to the grammatical structure of the sentences that express them. We shall cash this notion here by adopting the view that complex PRPs are best thought of as being “built up” logically from logically less complex constituents by means of a group of logical functions. These operators can be grouped into three classes. The first class consists of logical functions corresponding to the class of familiar syntactic operators from modal predicate logic; viz., the modal and boolean operators and the quantifiers. So, for instance, the property \(\lambda x \sim E!x\) nonexistence is the negation of the property of existence; i.e., \(\lambda x \sim E!x\) = \(\neg((\lambda x E!x))\) (note we are using rather than mentioning our abstracts here); the property \(\lambda x 0 \sim E!x\) possible nonexistence is the possibilization of \(\lambda x \sim E!x\); the relation being objects \(x\) and \(y\) such that \(x\) is rich and \(y\) is unhappy is the conjunction of the properties being rich and being unhappy; i.e., \(\lambda x(y Rx \land \sim H y) = \text{conj}(\forall x Rx), (\forall x H y))\); and the property being married (to someone) is the existential quantification of the second argument place of the marriage relation; i.e., \(\lambda x \exists y M xy\) = \(\exists y((\lambda x M xy))\).

Logical functions in the second class reflect certain possible relations between the \(\lambda\)-bound variables in an abstract \([\lambda v_1, \ldots, v_n]\) and occurrences (or lack thereof) of those variables in \(\varphi\). Though important in a complete account, these functions are irrelevant for our present purposes, and so we won’t examine them any further. Not so our final class, the predication functions. These are the functions that give rise most directly to singular propositions. These operators fall naturally into two camps: “simple” predication functions that are reflected in abstracts like \('[\lambda x \sim E!x]q]\) and \('[\lambda x B x(\lambda S x)]\) in which no term contains a free occurrence of any (top level) \(\lambda\)-bound variable; and “complex” predication functions reflected in abstracts like \('[\lambda x B x(\lambda S x)]\) and \('[\lambda x \sim E!x]q]\) in which there is such a term. As it happens, both simple and complex functions are instances of a single, general type. For purposes here, however, we can avoid the complications of the complex operators and think only in terms of their simpler brethren.

In the simplest case, a predication function \(\text{pred}\) takes a single object \(a\) and “plugs” it into the \(n\)th argument place of a property or relation, yielding a PRP \(R\) with one fewer argument place, and with \(a\) as a new constituent. Thus, in particular, in the case of a property the result is a singular proposition about \(a\). So, for example, \(\text{pred}\), takes the property of possible nonexistence—\(\lambda x 0 \sim E!x\)—and plugs Quine, say, into its first (and only) argument place, thereby yielding the proposition that Quine is possibly nonexistent; i.e., more formally, \(\text{pred}((\lambda x 0 \sim E!x)q) = (\lambda x 0 \sim E!x)q\). In general, however, a simple predication can involve an \(n\)-place relation and any of its argument places; consider, for example, such multi-place predications as the property \(\lambda x S a x b\) of being an \(x\) such that \(a\) says \(x\) to \(b\). To capture this, let \(\sigma = (i_1, \ldots, i_n)\) be any finite, increasing sequence of numbers greater than 0. The idea, then, is that, for any such \(\sigma\), we define a predication function \(\text{pred}_{\sigma}\) such that for any \(n \geq i_{\omega}\) if \(R\) is an \(n\)-place relation, and \(a_1, \ldots, a_m\) any \(m\) objects, then \(\text{pred}_{\sigma}(R, a_1, \ldots, a_m)\) is the predication of \(R\) of \(a_1, \ldots, a_m\) relative to the \(i_1, \ldots, i_n, i_m\) argument places of \(R\), respectively; that is, to express this using our language, \(\text{pred}_{\sigma}(R, a_1, \ldots, a_m) = ((\lambda x_1 \ldots x_{i_{\omega}} \ldots x_{i_m} \ldots x_{i_n} \ldots x_{i_{m+1}} \ldots x_{i_{m+1}} \ldots x_{n-m}) R x_1 \ldots x_{i_1} a_1 x_{i_1+1} \ldots x_{i_m} a_m x_{i_m+1} \ldots x_{i_{m+1}} \ldots x_{n-m})\). Thus, in particular, \(\text{pred}_{\sigma}(R, a, b) = (\lambda x S a x b)\).

Henceforth we will assume a variety of axioms that guarantee the fine-grainedness of PRPs. It is rather tedious to lay the groundwork for stating these axioms precisely. They can, however, be expressed with sufficient clarity and completeness for purposes here rather easily. First, it is assumed that \(P \neq Q\), where \(P\) is an \(n\)-place relation, and \(Q\) is an \(m\)-place relation, and \(n \neq m\). Next, it is assumed that the classes of modal, boolean, quantified, and predicative PRPs are all pairwise disjoint, where a modal PRP is one that is in the range of the possibilization or necessitation function, a boolean PRP is in the range of one of the boolean functions, and so on. This axiom guarantees, for instance, that the proposition \(\lambda x 0 \sim E!x)q\) Quine is possibly nonexistent, though logically equivalent to it, is distinct from the proposition \(\lambda 0 \sim E!x)q\) Possibly, Quine does not exist. The latter is a modal proposition, derived by applying the possibilization operator to the proposition that Quine does not exist, while the former is predicative, derived as noted above. The final group of axioms captures the most intuitive element of fine-grainedness; viz., that PRPs built up from different constituents must themselves differ. Thus, from these axioms it follows, for example, that the result of predicating the property being a philosopher of Quine, \(\lambda P q\), is a different proposition than the one, \(\lambda P q\), that results from predicating that property of Peter Geach, say, or of predicating the property existence of Quine, \(\lambda E!q\).
THE PUZZLE REVISITED

Now that we have an appropriate representational medium, let us use it to express the puzzle. We will do so by laying out the ordinary language rendering of the puzzle in its propositional form explicitly, following each premise with its formalized counterpart. Our formal language has no counterfactual conditional \( \Rightarrow \), and hence we can’t represent the subjunctive element of the argument precisely. However, since all the subjunctives are necessary, and since \( \Box(\phi \Rightarrow \psi) \) is logically equivalent to \( \Box(\phi \supset \psi) \),\(^9\) we can substitute ordinary entailment instead without affecting the logic of the argument. Also, as noted, for readability, we let \( T[\phi] =_y [\phi] \). Call the following argument RP.

(1) Quine might not have existed. (Assumption.)
(1') \( \Box \neg \text{Elq} \).
(2) Possibly, the proposition [Quine does not exist] is true. (From (1).)
(2') \( \Box \neg T[\neg \text{Elq}] \).
(3) Necessarily, if Quine hadn’t existed, [Quine does not exist] wouldn’t have existed. (By OD.)
(3') \( \Box (\neg \text{Elq} \supset \neg T[\neg \text{Elq}]) \).
(4) Necessarily, if [Quine does not exist] hadn’t existed, it wouldn’t have been true. (Assumption.)
(4') \( \Box (\neg T[\neg \text{Elq}] \supset \neg \text{Elq}) \).
(5) Hence, necessarily, if Quine hadn’t existed, [Quine does not exist] wouldn’t have been true. (From (3) and (4).)
(5') \( \Box (\neg \text{Elq} \supset \neg T[\neg \text{Elq}]) \).
(6) Necessarily, if Quine does exist, [Quine does not exist] is not true. (By logic.)
(6') \( \Box (\neg T[\neg \text{Elq}] \supset \text{Elq}) \).
(7) Hence, necessarily, whether Quine had existed or not, [Quine does not exist] would not have been true. (From (5a) and (6a).)
(7') \( \Box (\text{Elq} \vee \neg \text{Elq} \supset \neg T[\neg \text{Elq}]) \).
(8) Necessarily, either Quine exists or he doesn’t. (By logic.)
(8') \( \Box (\text{Elq} \vee \neg \text{Elq}) \).
(9) It is not possible that [Quine does not exist] be true, contradiction. (From (7) and (8), contradicting (2).)

(9') \( \neg T[\neg \text{Elq}] \).
(10) Therefore, it is false that Quine might not have existed. (From (2) and (9), discharge (1).)
(10') \( \neg \Box \neg \text{Elq} \).

THE UNDERLYING LOGIC

Before examining prospective solutions to the puzzle, it will be useful to make explicit the central logical principles underlying the argument. First, the move from (1) to (2), though intuitive, is not trivial. It is based, of course, upon the left-to-right direction of the intuitive equivalence \( \neg \text{Elq} \supset \neg T[\neg \text{Elq}] \), which is an instance of the general schema \( \Box \phi \equiv \Box T[\Box \phi] \). This is a modalized version of the principle \( \phi \equiv T[\Box \phi] \) that, roughly a statement holds if and only if the proposition it expresses is true. This principle in tum—bearing in mind our stipulation that \( T[\phi] \) is nothing other than \( [\phi] \)—is just a special case of \( \lambda \)-conversion:

\[
\lambda \text{-con: } \phi \equiv [\lambda x \phi] \tau,
\]
where \( x \) is an \( n \)-tuple of pairwise distinct variables and \( \tau \) an \( n \)-tuple of terms. So at least the left-to-right direction of \( \lambda \)-con appears to be one of the general logical principles underlying the argument. How then do we get from the left-to-right instance \( \neg \text{Elq} \supset T[\neg \text{Elq}] \) to its modalized counterpart \( \neg \text{Elq} \supset \Box T[\neg \text{Elq}] \)? The easiest route would seem to be this. From \( \neg \text{Elq} \supset T[\neg \text{Elq}] \), assuming the rule of Necessitation

\[
\Box \phi \equiv \Box [\lambda x \phi] \tau,
\]
we have \( \Box (\neg \text{Elq} \supset T[\neg \text{Elq}]) \). From the basic modal propositional schema

\[
\begin{align*}
\text{K: } & \Box (\phi \supset \psi) \supset (\Box \phi \supset \Box \psi) \\
\text{\Box \phi: } & \Box \phi \equiv \neg \Box \neg \phi,
\end{align*}
\]

some propositional logic, and the usual principle that \( \Box \) and \( \neg \) are interdefinable, i.e.,

\[
\Box \phi \equiv \neg \Box \neg \phi,
\]
we can derive the schema \( \Box (\neg \phi \supset \psi) \supset (\neg \phi \supset \Box \psi) \). Thus, plugging in \( \neg \text{Elq} \) and \( T[\neg \text{Elq}] \) for \( \phi \) and \( \psi \), respectively, we have \( \neg \text{Elq} \supset \Box T[\neg \text{Elq}] \), as desired.

Now already at this early stage of the game difficulties begin to loom. It is, presumably, a logical truth that \( \text{Elx} \equiv 3y(y = x) \). Hence, because \( \forall x \exists y(y = x) \) is a theorem of classical quantification theory (CQT) with identity, \( \text{Elq} \) is as well. So in the context of CQT with identity, \( \Box \) and \( \Box \neg \) alone are enough to yield (10'). Exploring this problem now will get us ahead of ourselves. So for the moment we will simply assume some reasonable fix, e.g., switching from CQT to a free quantification theory (i.e., a quantification theory that allows nondenoting terms and hence does not have \( \text{Elx} \) as a theorem).
Next, it would be well to make the principle of ontological dependence—of which (3) is an instance—explicit; in fact, let us state a slightly more general principle that captures the idea that the constituents of a PRP are also jointly sufficient for its existence as well as individually necessary. Where \( \tau = \tau_1 \ldots \tau_n \) is a nonrepeating sequence of terms, let \( E!'\tau \) abbreviate \( E!'\tau_1 \land \ldots \land E!'\tau_n \); then the principle in question is:

\[
\text{Ex: } \ E![(\forall \ x \ \varphi)] = E!'\tau,
\]

where \( \tau \) contains all and only the noncomplex terms (i.e., constants, variables, and primitive predicates) that occur free in \( [\forall \ x \ \varphi] \).

Though not strictly required by the proof, the right-to-left direction is clearly a part of the underlying metaphysics, and hence we will have to hold in a complete logic for it. Note also that, given Necessitation, we need not explicitly modalize the principle. The principle is called \text{Ex} because it is a reasonably precise statement (within the expressive boundaries of our language) of the doctrine that has come to be known (for better or worse) as existentialism.¹⁰

Existentialism is sometimes characterized as the view that the only PRPs that exist necessarily are those that are either purely qualitative—i.e., roughly, those that count no concrete individuals among their constituents—or whose concrete constituents themselves all exist necessarily. Though not strictly entailed by existentialism proper—i.e., in the form expressed by \text{Ex}—the view is a natural correlate. Since identity and existence are generally taken to be purely qualitative, to capture this view in our logic we must express their necessary existence explicitly. To simplify matters we will identify existence, \( E! \), with the property \( [\forall x \exists y(x = y)] \) of being identical with something. Then we need only add the axiom:

\[
E!'\vdash E!=-.
\]

The necessary existence of the identity relation now follows from \( E!=- \) and \( \Box I \), and that of existence from \text{Ex}, \( \Box I \), and \( K \).¹¹

Finally, let us unpack the important assumption (4) a little more explicitly. (4) is an instance of the principle that a proposition cannot be true without existing:

\[
(11) \ T[\varphi] \vdash E![\varphi].
\]

or more generally still, the principle used in the general form of the Russellian puzzle that a PRP cannot be exemplified without existing (where for a proposition to be exemplified is just for it to be true):

\[
(12) \ \pi \tau \vdash E!'\pi,
\]

where \( \pi \) is any \( n \)-place predicate and \( \tau \) a sequence of \( n \) terms. Intuitively, (12) expresses a class of instances of what Plantinga has called serious actualism, the view that an object cannot have a property or stand in a relation without existing:

\[
\text{SA: } \ \pi \tau \vdash E!'\pi.
\]

For, intuitively, from \( Pt_1 \ldots t_n \) it follows that \( P \) stands in the exemplification relation \( \lambda \) with \( t_1 \ldots t_n \Delta Pt_1 \ldots t_n \) (or where \( n = 0 \), that \( \pi \) has the property of being true), and hence that \( E![P] \) by \text{SA}. However, because truth and exemplification are not expressible PRPs in our language, nothing equivalent to either (11) or (12) follows in this manner from \text{SA}. Given \text{Ex}, however, all three statements follow from the generalized serious actualism principle:

\[
\text{GSA: } \ \varphi \vdash E![\varphi], \text{ where } \varphi \text{ is atomic;}
\]

i.e., roughly, an assertion cannot hold unless it expresses an existing proposition, and hence by \text{Ex} unless every constituent of the expressed proposition exists. In particular, we have as a theorem a succinct expression of (12) and \text{SA}:

\[
(13) \ \pi \tau \vdash (E!'\pi \land E!'\tau).
\]

Note also that by \text{GSA}, \text{Ex}, and \( \lambda \text{-con} \) we have in general:

\[
(14) \ \varphi \vdash E![\varphi] \text{ for any } \varphi.
\]

For by \( \lambda \text{-con} \) we have \( \varphi \vdash [\varphi] \), and so since \( [\varphi] \) is a 0-ary atomic formula, we have \( [\varphi] \vdash E![\varphi] \) by \text{GSA}. Several applications of \text{Ex} in both directions and some propositional logic yield the desired result.

I will take \text{GSA} to be the general metaphysical principle underlying (4).

**AN ACTUALIST ACCOUNT OF POSSIBLE WORLDS**

In our search for a solution to the Russelian Puzzle, it will be illuminating to make use of the familiar notion of a possible world. Possible worlds are of course a controversial sort of entity, and it will be important for our discussion that whatever account of worlds we adopt be actualistically respectable. Familiar accounts from the likes of Adams, Plantinga, Pollock, and others take worlds to be (sets of) propositions or states of affairs.¹² I find these accounts problematic, and so will use instead a more model-theoretic account of possible worlds that takes them to be set-theoretic constructions exhibiting the structure of things as they might have been.

As a first cut at this account, consider a set \( D = \cup \{D_1, D_0, D_1', \ldots \} \), where \( D_1 \) is a set of individuals (i.e., non-PRPs) and, for \( n \geq 0 \), \( D_n \) is a set of \( n \)-place relations that includes existence \( E! \) in \( D_1 \). Let \( ext \) be an extension function that maps the members of \( D_0 \) into truth and falsity, the members of \( D_1 \) into subsets of \( D \), and the members of \( D_n \) for \( n > 0 \) into \( n \)-tuples of members of \( D \). We stipulate in particular that \( ext(E!) = D \). Then say that the pair
$S = (D, ext)$ is a state of affairs just in case the actual extensions of the PRPs in D, restricted to D, are exactly as ext depicts them, and say that S is a possible state of affairs just in case it is possible that S be a state of affairs; i.e., just in case it is possible that all the members of D exist (together) and that ext correctly depicts the extensions of the PRPs in D. Note that since it is constructed out of actually existing objects, every possible state of affairs S exists, even if it is not actual; i.e., even if ext does not depict the actual extensions of the PRPs in S. We assume in addition that possible states of affairs exhibit our underlying Russellian metaphysics with respect to the PRPs they contain as embodied in the principle Ex, so that an n-place PRP P $\in D_i$ if all its constituents are in D. Given this, note also that since ext maps PRPs into D, each possible state of affairs is one in which the generalized serious actualism principle GSA holds. Finally, say that S is a possible world, or world structure, just in case, among the possible states of affairs, it is possible that S be the largest. That is, let A be the class of possible states of affairs; then S is a possible world just in case, possibly, S is a state of affairs and for any $S' = (D', ext') \in A$, if $S'$ is a state of affairs, then $D_i' \subseteq D_i$ for all i.14

A limitation of this construction arises from the fact that, intuitively, there could have been things other than the things that actually exist; John Paul II, for example, under quite different circumstances, might have had a grandson. What possible state of affairs represents this possibility? Though we will not explore this issue in detail in this paper, the crucial idea is to set aside some class of actualistically acceptable objects—pure sets, say—to serve as surrogates for such "possibilities" as John Paul II's grandson. These, in turn, can serve as constituents of "surrogate PRPs"; that is, PRPs containing surrogates among their constituents that therefore serve as surrogates for PRPs that would exist if the individuals represented by those surrogates existed. Thus, if the empty set $\emptyset$ were to be a surrogate for a grandson of John Paul II in some possible state of affairs, then the proposition [$\emptyset$ is sitting] will represent the corresponding proposition that would exist if that grandson were to exist.15 A possible state of affairs, then, is a pair $S = (D, ext)$ such that it is possible for there to be some mapping $m$ from the surrogates in D into the set of existing objects16 such that, under that mapping, ext correctly depicts the extensions in D of the PRPs (or surrogates thereof; note that on our type-free picture the PRPs form a subset of D). The following figure illustrates the idea in a simple fashion. Where $S = (D, ext)$, the picture indicates a set $D_1$ of properties and a set $D_1$, consisting of five individuals, three of which are surrogates, including $\emptyset$. A portion of ext is indicated by the lines from three properties in the set of PRPs to subsets of $D_1$. M indicates the set of individuals that would have existed were John Paul II to have had a grandchild, and $m$ is the mapping there would have been from $D_1$ into M; in particular, $m$ maps John Paul II to himself and $\emptyset$ to his grandchild.

A possible world, then, is a possible state of affairs that is possibly the largest state of affairs in the sense defined above (except with 'state of affairs' understood as in the current paragraph, of course). The important point to note is that by adopting an actually existing model-theoretic structure to play the role typically accorded to possible objects and possible worlds, we can use the often helpful language of possibilities without committing ourselves to an ungainly possibilist metaphysics.

**SOLUTION 1: PRIOREAN INTERNALISM**

The first solution to the puzzle I want to consider is suggested by the work of Arthur Prior.17 Prior's solution is, at first sight anyway, extreme. While the logic underlying the Russellian puzzle is flawed, nonetheless, if we take the implications of the nature of singular propositions seriously, its conclusion is still unavoidable. As Prior notes:
If we so interpret ‘it could be that φ’ that it is true if and only if [φ] could be true, then my non-existence is something that could not be, since [I do not exist] is not a thing that could be true. So, for Prior, there are two related problems with the underlying logic of the Russellian puzzle. The first is that from the true conclusion that Quine’s nonexistence is impossible, it follows by □◊ that his existence is necessary, that he is bound to exist. A similar argument, of course, applies to every ostensibly contingently existing thing, and that, Prior notes, “makes gods of us all.” □◊ is therefore an inappropriate logical principle in a modal logic for contingent beings.

The problems don’t stop there, however. Consider next the following argument. From the propositional tautology

\[(15)\] \[E!q \lor -E!q\]

it follows from λ-con that the proposition expressed is true,

\[(16)\] \[\top \vdash E!q \lor -E!q\].

As an instance of (11), we have

\[(17)\] \[\top \vdash E!q \lor -E!q \supset E![E!q \lor -E!q]\],

by Ex we have

\[(18)\] \[E![E!q \lor -E!q] \supset E!q\],

and hence by propositional logic

\[(19)\] \[\top \vdash E!q \lor -E!q \supset E!q\],

and so by Necessitation and K we have

\[(20)\] \[\top \vdash E!q \lor -E!q \supset \Box E!q\].

By Necessitation once again, from (16), we also have

\[(21)\] \[\Box E!q \lor -E!q\],

and so by Modus Ponens, the necessity of Quine’s existence falls out once again:

\[(22)\] \[\Box E!q\].

It is no mystery what the culprit is here. Since (15) is a tautology, (16) should be a theorem in any logic with terms denoting the propositions expressed by its sentences and some means of expressing truth; for if a sentence expresses a logical truth, how could it not also be a truth of logic that the proposition so expressed is true? That, in the 0-place case, is just what λ-con guarantees. However, although it is a truth of logic, Prior argues, (15) is not a necessary truth; i.e., a proposition that would have been true no matter what. For by our underlying metaphysics, it appears that that could be so only if it were a necessary being, and hence only if all of its constituents, Quine in particular, were necessary. As Prior himself remarks in regard to a similar logical truth, “if it is necessary that if I am a logician then I am a logician, it is necessary that I am.” Necessitation, in the guise of □I, is therefore no more appropriate than □◊.

Prior is thus what we might term an internalist: In terms of the apparatus above, the only basis for evaluating the truth of a proposition P with respect to a possible world w = ⟨Dw, extw⟩ is its truth-value in the world; i.e., the value of extw on P. Because extw is defined only on the PRPs in w, it follows that a proposition can be evaluated as true (or false) only with respect to those worlds in which it exists. Let us make this more explicit by drawing out the connections between a proposition’s logical form and its extension. For Prior, in accordance with GSA, the following principle governs the atomic case:

\[P1\] \[extw(\{P \lor \ldots \lor P\}) = \top \text{ just in case (i) } n = 0 \text{ and } extw(P) = \top, \text{ or (ii) } n > 0 \text{ and } \langle a_1, \ldots, a_n \rangle \in extw(P).\]

That is, for an atomic proposition \[P \lor \ldots \lor P\] when \(n = 0\) to be considered true in \(w, a_1, \ldots, a_n\) must stand in the relation P (or, in the 0-place case, P must be true) in w, and hence, by the definition of extw, the arguments to the relation \(P \lor \ldots \lor P\) (hence, by Ex, P itself) must all exist in w. The cases of conjunctive and quantified propositions work as expected. The negated and modal cases are worth considering overtly:

\[P2\] \[extw(\{\neg φ\}) = \top \text{ just in case } extw(\{φ\}) = \bot]\.

Given the definition of extw once again it follows that a negated proposition must both exist in a given world and be such that its unnegated counterpart is false there in order for it to be true in the world; this is crucial to the Priorian picture.

Since [] and ◊ are not interdefinable we need clauses for each of them:

\[P3\] \[extw(\{[α]\}) = \top \text{ just in case } extw(\{[α]\}) = \top \text{ for some possible world } w;\]

\[P4\] \[extw(\{[α]\}) = \top \text{ just in case } extw(\{[α]\}) = \top \text{ for all possible worlds } w;\]

Given the definition of extw it is clear that, in accordance with P1, the proposition that Quine does not exist, \([-E!q]\), is not true in any world, and hence that \([-◊-E!q]\) is true. But, as expected, \([\Box E!q]\) is not true, since \([-E!q]\) does not exist in any world lacking Quine, and thus ◊ fails. And since \([E!q \lor -E!q]\) does not exist in any such world as well, □ fails as well. P1–P4 thus appear to capture the truth conditions for propositions in Prior’s view.

If, as just noted, the conclusion of RP is sound for Prior, what is an acceptable argument for it? The best way to answer is to think about how the logic above needs to be revised in light of the failure of □◊ and □I. Let us note first that it is only one direction of □◊ that breaks down, the implication from ¬◊φ to □φ; as we’ve just seen, from the fact that the proposition
[φ] could not possibly have been false it doesn’t follow that it would therefore have been necessary; i.e., counted among the true propositions that exist in every world. However, equally surely, if [φ] is necessary, if it would have been among the true propositions no matter what, then it could not possibly have been false. ~O¬φ thus expresses a sort of weak necessity, implied by, but not implying, its full-blooded counterpart. To simplify notation, then, let us introduce a weak necessity operator explicitly:

\[ \Dia \Box \phi \equiv \Box \neg O \neg \phi, \]

and nail down its relation to its stronger counterpart:

\[ \Box \Box \phi \equiv \Box \phi \supset \Box \phi. \]

What about the other direction? As just noted, what prevents a weakly necessary proposition like [E!q \lor \neg E!q] from being strongly necessary is the fact that it is not a necessary being. It is (loosely speaking) true \textit{whenever it exists}—that’s just what it is to be weakly necessary—but it is not the case that it would have been true no matter what. Thus, for a proposition to be strongly necessary it must be both true whenever it exists, and furthermore such that it never fails to exist; i.e., a little more precisely, it must be both weakly necessary and necessarily existent:

\[ \Box \Box \phi \equiv \Box \phi \land \Box E!(\phi) \supset \Box \phi. \]

Both conditions can be combined into the single equivalence

\[ \Box \Box \phi = \Box \phi \land \Box E!(\phi). \]

The reasoning behind \Box \Box also determines the fate of Necessitation: Because of contingently existing logical truths, we are warranted only in inferring that the weak necessitation of an arbitrary logical truth is a logical truth. Hence, in our Priororean system, \Box I is simply replaced by its weak counterpart:

\[ \Box I: \Box \phi \Rightarrow \Box \Box \phi. \]

To infer more, we have to know more about the ontological status of the proposition \[\phi\], as captured precisely in the derived rule (from \Box \Box and \Box I)

\[ \text{DRC}: \Box \phi \Rightarrow \Box \Box E!(\phi) \supset \Box \phi; \]

i.e., the necessitation of a \textit{necessarily existing} logical truth is itself a logical truth.

This raises the question of how one proves that a given proposition \(P\) exists necessarily. Generally, this is accomplished by showing or assuming that all of its constituents exist necessarily. Thus, one appeals to (the right-to-left direction of) a modalized version of Ex:

\[ \Box \Box \text{Ex}: \Box \Box E!(\exists x \phi) = \Box \Box E! \tau, \]

where \(\tau\) is as in Ex. \Box \Box Ex is clearly true in our underlying metaphysics. However, with strong necessitation replaced by weak necessitation, it is no longer possible to prove it from Ex. Hence, \Box \Box Ex too needs to be added as a separate logical principle in its own right.

Finally, though \(K\) remains true in this Priororean logic, it is of limited use due to its applicability only to necessarily existing propositions. Hence, we need a corresponding principle to govern weak necessity. We cannot, however, simply replace \Box with \Box. Consider, for example, the conditional “If Quine is human, then something is human,” \(Hq \supset \Box \exists Hx\). This proposition is weakly necessary for Prior: In terms of the usual jargon, in every world in which it exists, the proposition that if Quine is human, something is human is true. But now suppose we add \[\Box (\phi \supset \psi) \supset (\Box \phi \supset \Box \psi)\] as an axiom. Then it follows that \[\Box Hq \supset \Box \exists Hx\]. It is indeed plausible that \[\Box Hq\]; i.e., in effect, that Quine is essentially human. However, it is false that \[\Box \exists Hx\]; it surely could have been that there were no humans at all.

Where things go wrong is that, unlike the case of \(K\), \([\psi]\) can exist without \([\phi]\), and in such circumstances its truth is not guaranteed. So what is needed is a simple qualification that rules such circumstances out:

\[ \text{K}: \Box (\phi \supset \psi) \supset (\Box \phi \supset \Box (E! \tau \supset \psi)), \]

where \(\tau\) contains all the noncompositional terms that occur free in \(\phi\) but not \(\psi\).

(Where \(\tau\) is null, we take instances of \text{K} to be of the same form as \(K\), only with \Box replacing \Box.) Returning to our example, all that follows now from \[\Box (Hq \supset \Box \exists Hx)\] and \[\Box Hq = \Box (E!q \supset \exists Hx)\], and that is unproblematic. For in any situation in which Quine exists he is human (by assumption), and so in any situation in which he exists something is human, as required. It should be noted that the original principle \(K\) now follows from \text{K} and the other principles above.

Now, regarding the Russellian puzzle, if we simply change every \Box to \Box, \text{RP} is in fact valid in our Priororean logic. However, we need not go to such lengths. For an advantage of the logic, with its weaker brand of necessitation, is that we are free to return to CQT. Thus, we can infer Quine’s existence, \(E!q\), directly, and hence by \Box I the impossibility of his nonexistence, \Box \neg E!q, but not thereby his necessary existence, \Box E!q, in accordance with Prior’s initial conception.

Now, this is perhaps a serviceable, if not particular comely, quantified modal logic, and it certainly appears to capture the metaphysics behind \text{RP} with great precision. However, something is still amiss. For surely there is a sense in which (1) is true; surely it is possible in some sense that Quine fail to exist. And indeed Prior agrees:22

\[ \ldots \text{there is a sense of ‘This might not have existed’ in which what it says could be the case (and generally is), i.e., the sense: ‘It is not the case that (it is necessary that (x exists))’} \]
To account for the intuitive truth of (1), then, corresponding to the weak necessity operator $\square$. Prior in effect introduces a weak possibility operator $\Diamond$:

$$\text{Def} \Diamond: \quad \Diamond \varphi \equiv_{df} \neg \square \neg \varphi.$$ 

Quine’s nonexistence, therefore, while not strongly possible, is nevertheless weakly possible, and that accounts for our intuition that (1) is true. Prior thus appears to be able to have it both ways: The metaphysics of singular propositions dictates that (1) must be false; semantical intuitions tell us it is true. The two senses of possibility appear to let us hang on to both the demands of metaphysics and the appeals of intuition.

But not so. Consider a logically false singular proposition; that, say, Quine both is and is not a logician, $[Lq \land \neg Lq]$. Since $[Lq \land \neg Lq]$ contains Quine among its constituents, it is not itself a necessary being. Hence, though false whenever it exists, the proposition fails to be false in situations in which it does not exist, and hence it is not necessarily false. It is thus a theorem of our Priorian logic that $\neg \square (Lq \land \neg Lq)$; i.e., it is a theorem that the contradiction in question is weakly possible, $\Diamond (Lq \land \neg Lq)$. Thus, in this logic, Quine’s nonexistence is possible in precisely the same sense that his being both a logician and a nonlogician is possible. But surely the former is true in some sense that the latter is not; surely, in some sense, Quine’s nonexistence is a way things could have been and his simultaneous existence and nonexistence is not. Prior’s logic, however, is unable to distinguish them, and hence fails to explain our intuition that (1) is true. Can we do better and still remain within the bounds of actualism?

**SOLUTION 2: ADAMS’ PERSPECTIVALISM**

Perhaps we can. In his seminal article “Actualism and Thisness,” Robert Adams suggests an actualist understanding of modal propositions that does not generally cash their truth conditions in terms of what propositions could or must have been true in Prior’s sense. In particular, on Adams’ view, the possibility of his nonexistence is not a matter of the truth of the proposition [Adams does not exist] within some possible world. He writes:

"... I deny ... that it is possible that $p$ always implies that the proposition that $p$ could have been true. Philosophers have often found it natural to characterize possibilities and necessities in terms of what propositions would have been true in some or all possible situations. ... This seems harmless enough so long as it is assumed that all propositions are necessary beings. But it is misleading if (as I hold) some propositions exist only contingently."
...‘I am a non-fish’ means that I am something that is not a fish; it ascribes to me the property of being a non-fish. If I did not exist, might I have that property? Might I be something that is not a fish? No, I would be nothing at all, and would have no properties. Hence ‘I am a non-fish’ is appropriately counted false in worlds in which I do not exist.27

However, to deny Adams’ non-fishiness at worlds w in which he doesn’t exist, i.e., to deny that he is in the extension of $[\lambda x \neg Fx]_w$ in w and hence to affirm that the atomic proposition $[\lambda x \neg Fx]_w$ is false at w, is not to deny that Adams is not a fish at those worlds; that is to say, while $[\lambda x \neg Fx]_w$ is false at w, the negated proposition $\neg [F]_w$ is true at w, and indeed at any world: It is true in, hence at, those worlds in which Adams exists (assuming he is essentially non-fishy) and at those worlds in which he fails to exist simpliciter. Either way he is not among the fishy things. More generally, then,

A2 $[\neg \phi]_w$ is true at w just in case $[\phi]_w$ is not true at w.

Unlike Prior, then, the concept of truth at a world gives us a sense of truth on which negated propositions can turn out to be true with respect to the worlds in which they don’t exist. This is the sense in which such propositions can nonetheless be counted possible. That is, more generally:

A3 $[\phi]_w$ is true at w (the actual world, in particular) just in case there is a world w’ such that $[\phi]_w$ is true at w’.

From A2 and A3 in particular it follows that a proposition of the form $[\neg \neg \phi]_w$ is true at w iff $[\neg \phi]_w$ is not true at w iff it is not the case that there is a world w’ such that $[\phi]_{w’}$ is true at w’; i.e., iff $[\phi]_w$ is true at all worlds w’. Since $[\phi]_w$ need not in general exist at a world to be true at it, it follows that a proposition can be true at all worlds without being true in all worlds. In particular, singular truths of propositional logic are true at all worlds. Consider, for example, the proposition $[Lq \vee \neg Lq]$ that Quine is either a logician or not a logician. This proposition is true at a given world w just in case either $[Lq]$ is true at w or $[\neg Lq]$ is. Suppose Quine exists in w. Then both $[Lq]$ and $[\neg Lq]$ exist there as well, and obviously one or the other is true; i.e., the proposition is weakly necessary. However, suppose Quine doesn’t exist in w. Then $[\neg Lq]$ is still true at w, since $[Lq]$ is not. Taking necessity to be truth at all worlds, then, it follows that $[(Lq \vee \neg Lq)]$ is necessary; i.e., the proposition $[(Lq \vee \neg Lq)]$ is true. We therefore have the original scope of strong necessity returned to us intact, and this, in turn, signals a return to our original modal principles $\Box\phi$ and full necessitation $\Box\Box$ (and hence also suitable modifications to CQT, for reasons noted above).

We can now see clearly what lies behind Adams’ denial that “It is possible that p” always implies that the proposition that $\neg p$ could have been true.” Note first that as a special case of A1, where P is a proposition $[\phi]$, we have that the proposition $[\neg \phi]$ is true at w just in case $ext_{\neg\phi}(\phi)_w = \top$. This reads a bit more intuitively when we use our pseudo-truth predicate: $[\neg \phi]_w = \top$ is true at w just in case $ext_{\neg\phi}(\phi)_w = \top$; i.e., the proposition that $[\phi]_w$ is true is itself true at w just in case the proposition $[\phi]_w$ is true in w. Either way, the important point to notice is that $[\neg \phi]_w$ (in contradistinction to $[\phi]_w$ in general) is the proposition expressed by a (0-place) predication, and hence, like all predications for the serious actualist, the components of $[\neg \phi]_w$, hence $[\neg \phi]_w$ itself, must exist in a world w in order for it to be true at w.

So consider now any property that Quine has essentially; existence is the obvious example. Then because there are worlds in which Quine doesn’t exist, the principles above entail that it is possible that Quine fail to exist, $\neg E!q$; i.e., the proposition $[\neg E!q]_w$ that this formula expresses is true (i.e., true at the actual world): $[\neg E!q]_w$ is true iff $[E!q]_w$ is true at some world w (by A3) iff $[E!q]_w$ is not true at some world w (by A2) iff there is a world w, such that $q \not\in ext_{\neg E!(\neg \neg E!q)}$, as we’ve supposed. Consider by contrast the claim $[\neg E!q]_w$ that the proposition $[\neg E!q]_w$ could have been true. This claim is true iff the proposition $[\neg E!q]_w$ it expresses is true iff there is a world w such that the atomic proposition $[\neg E!q]_w$ is true at w (by A3) iff $ext_{\neg E!(\neg \neg E!q)}(\neg E!q) = \top$ (by A1); i.e., iff the proposition $[\neg E!q]_w$ is true in w, and hence only if $q \in D_w = ext_{\neg E!(\neg \neg E!q)}$, by our construction. This of course can’t be, since w is a possible state of affairs. So while it is possible that Quine fails to exist, $\neg E!q$, the proposition $[\neg E!q]_w$ that he does not could not be true; i.e., $\neg [\neg E!q]_w$. Thus, to tighten up Adams’ remark above, on this approach we distinguish the truth of a proposition to the effect that a given proposition P is possible from the possibility of P’s being true.

The argument generalizes to properties and relations: We can show in the same fashion that, e.g., even though it is possible that Quine fail to exist, $\neg E!q$, it does not follow that it is possible that he be nonexistent, that he exemplify the property of being nonexistent, $\Box [\neg E!q]$. For this is so iff the proposition $[\neg E!q]_w$ that it is possible that Quine be nonexistent is true iff there is a world w such that the atomic proposition $[\neg E!q]_w$ that Quine is nonexistent is true at w iff $q \not\in ext_{\neg E!(\neg \neg E!q)}(\neg E!q)$ and hence only if $q \in D_w = ext_{\neg E!(\neg \neg E!q)}$ once again.

The general logical lesson here is that the law of $\lambda$-conversion appears to break down in modal contexts. That is, in particular, whereas both

(23) $[\neg E!q]_w = \neg E!q$

and

(24) $\lambda x [\neg E!x]q \equiv \neg E!q$

appear to be valid (as they ought, being straightforward instances of $\lambda$-conversion in which the modal operators play no essential role), both

(25) $[\neg E!q]_w = \neg E!q$

and
are false, the difference being that both sides of (23) and (24) require only that \([-E!q]\) be true at some world (given \(\Box \neg E!q\)), while the left sides of (25) and (26) require it to be true in some world. Currently, however, the latter two are provable in our revised system as it stands: In the case of (25), for instance, by \(\lambda\)-conversion and propositional logic we have

\[
\Box [\neg E!q] \equiv E!q,
\]

and so by \(\Box I\) we have

\[
\Box [\neg E!q] \equiv E!q,
\]

which creates the vacuous modal context, and (25) follows by \(K, \Box / \Box\), and contraposition.

The latter three principles are unimpeachable in the current context, so the problem appears to lie with either \(\lambda\)-conversion or necessitation. \(\lambda\)-conversion, however, seems scarcely more controversial than the principles above: In particular, in the \(0\)-place case used in (27), what else could it be for \(\varphi\) to hold than for the proposition it expresses to be true? In fact, we shall affirm the validity of \(\lambda\)-conversion, though as a theorem rather than an axiom; the problem lies more directly with necessitation.

To get at the nature of the problem and its solution, let us return to the problem of CQT in quantified modal logic. The problem, recall, is that \(E!x\) is a theorem of CQT (letting \(E!x \equiv \Box \exists y(y = x)\)), and so by necessitation it follows that \(\Box E!x\) is a theorem as well; i.e., it becomes a truth of logic that everything is a necessary being. A common way around this difficulty, as noted, is to move to a free quantification theory, particularly, to replace the usual universal instantiation axiom

\[
\Box I: \forall x \varphi \supset \varphi^x, \text{ for any term } \tau \text{ that is free for } x \text{ in } \varphi
\]

with its free counterpart

\[
\Box FUI: \forall x \varphi \supset (E!x \supset \varphi^x), \text{ for any term } \tau \text{ that is free for } x \text{ in } \varphi.
\]

From \(\Box FUI\), together with the reflexivity axiom for identity

\[
\Box Id: x = x,
\]

it is possible only to prove \(E!x \supset \exists y(y = x)\), i.e., \(E!x \supset E!x\), not \(E!x\) simpliciter, and so instead of necessitarianism we derive only the innocuous \(\Box (E!x \supset E!x)\); i.e., it is true at every world \(w\) that if \(x\) exists in \(w\), then \(x\) exists in \(w\); i.e., it is true at every world \(w\) that either \(x\) exists there or \(x\) doesn't exist there; i.e., that \([E!x] \in w\) or \([E!x] \notin w\). So in this respect \(\Box FUI\) appears to be just what we need. However, this solution is in one important way unlovely. The chief motivation behind free logic is to have a logical system capable of dealing with the possibility of nonnoting terms. That is (perhaps) all well and good; but as a solution to the problem of CQT in modal logic it foists undesirable baggage upon us. Following Prior, we are constructing an actualist modal logic of contingent beings, and hence we would like to have at our disposal at least the possibility of signaling this fact: by having \(E!x\) fall out as a theorem. \(\Box FUI\) thus gives us the right result in modal contexts, but is unduly restrictive in nonmodal contexts.

Fortunately, in this case at least, we can have our cake and eat it too. Consider \(\Box Id\) once again. If we take '=' to be a predicate that denotes a full-fledged relation (as, I think, we ought), then we do not want it to follow—as \(\tau\) does given necessitation—that \(\Box (x = x)\); i.e., that the proposition \([x = x]\) is necessarily true, for any \(x\). For otherwise, by (ii) in \(\Box A\) (and the definition of \(ext^\tau_n\)), it follows that \(x\) exists in every world. Equally clearly, however, we don't want to lose \(\Box Id\) as a logical truth; '\(x = x\)' is true under any interpretation. So this suggests a natural restriction on necessitation; viz.,

\[
\Box I': \Box \neg \varphi \supset \Box \neg \varphi, \text{ so long as } \varphi \text{ provable without any instance of } \Box Id.
\]

This of course prevents the inference from \(\Box Id\) to its necessitation. At the same time, however, together with \(\Box SA\), \(\Box Id\) yields \(E!x\) and hence, in conjunction with \(\Box FUI\), full \(\Box UI\). But since a proof of \(E!x\) requires \(\Box Id\), its necessitation is not provable. Thus, by a well-motivated (for actualists, anyway) restriction on necessitation we avoid the necessitarian problems of CQT without abandoning it.

We can frame a precisely analogous solution to the problem of \(\lambda\)-conversion. Consider first that the usual axiom can be broken down into two conditionals:

\[
\lambda_{\Box}: \lambda x \varphi y \supset \varphi^x
\]

and

\[
\lambda_{\Pi}: \varphi^x \supset \lambda x \varphi y,
\]

where \(x\) and \(y\) are nonrepeating sequences \(x_1, \ldots, x_n\) and \(y_1, \ldots, y_n\), respectively, and for all \(i \leq n\), \(y_i\) is free for \(x_i\) in \(\varphi\). Applying necessitation to instances of \(\lambda_{\Box}\) corresponding to (25) and (26), we have

\[
\Box [\neg E!q] \supset \neg E!q
\]

and

\[
\Box (\neg E!x q \supset \neg E!q),
\]

which are clearly unproblematic: (29) is true iff the proposition \([\neg E!q] \supset \neg E!q\) is true at every world \(w\) iff either \([\neg E!q] \in w\) is false at \(w\) or \([\neg E!q] \in w\) is true at \(w\) iff \(ext^\tau([\neg E!q]) = F\) or \([E!q] \in w\) is not true at \(w\) iff \(ext^\tau([E!q]) = T\) or \(q \notin ext^\tau_E\). (30) follows in
much the same way. Not so the relevant instances of $\lambda_{RL}$; viz.,

\[(31) \Box(-E! q \supset [-E! q])\]

and

\[(32) \Box(-E! q \supset [\lambda x -E! x! q]).\]

From the fact that $[-E! q]$ is true at a given world $w$, it certainly does not follow (and indeed, when ‘E!’ denotes existence, it cannot follow) that either $[[E! q]]$ is true at $w$; i.e., that $ext_q([-E! q]) = T$, or that $[[\lambda x -E! x! q]]$ is true at $w$; i.e., that $q \in ext_q((\lambda x -E! x! x))$. For both would require $q$’s existence in $w$ by our construction. The problem in general, of course, is that a negated atomic proposition $[-Pq]$ can be true at a world either through $q$’s existing in $w$ but failing to exemplify $P$, in which case both $[[Pq]]$ and $[[\lambda x -P! x! q]]$ are true at $w$ as well, or through $q$’s failing to exist there; in which case neither proposition is true at $w$.

What these observations suggest, then, is a qualification of $\lambda_{RL}$ analogous to the qualification of UI that yields FUI: An instance of the conditional $\lambda_{RL}$ holds in a given world $w$ so long as all the individuals referred to in the antecedent exist in $w$. That is, we replace $\lambda_{RL}$ with

$$\lambda_{RL}': \varphi \supset (E! \pi \supset [\lambda x \varphi]! y),$$

where $\pi$ contains the variables $y_i$ and all other noncomplex terms occurring free in $\varphi$, and where for all $i \leq n$, $y_i$ is free for $x_i$ in $\varphi$.

$\Box \pi$ can now be applied unproblematically to $\lambda_{RL}'$, but because of the restriction on its general applicability, (31), (32), and their troublesome ilk will no longer follow. But because $E! x$ is a theorem, full $\lambda$-conversion—though not its necessitation—is provable from $\lambda_{RL}$ and $\lambda_{RL}'$, as desired.

This then is the key to the solution of the Russellian puzzle on our reconstruction of Adams’ approach. As noted above, the move from (1) to (2) requires the application of necessitation to $-E! q \supset [-E! q]$ to yield (31) and thence $\varphi -E! q \supset \varphi -E! q$. But the deduction of $-E! q \supset [-E! q]$ in the above logic requires $\lambda_{RL}$, which requires $E! q$, which in turn requires $\Box \pi$. Thus, $\Box \pi'$ cannot be invoked to yield (31), and so the argument is blocked. We get only the innocuous, indeed propositionally trivial, $\Box(-E! q \supset (E! q \supset [-E! q])).$

SOLUTION 3: FULL PERSPECTIVALISM

GENERALIZING TRUTH-AT, I: RELATIONS

Preferable as this logic is to the Priorian logic above, however, it is still too much in the latter’s clutches. To see this, for a given notion of extension $e$, say that the extension of a proposition $P$ is positive with respect to a given world just in case $e(P, w) = T$. What the concept of truth-at gives us, in contrast to the Priorian view, is a notion of extension on which a proposition has a positive extension with respect to worlds in which it doesn’t exist. Using this distinction, as we’ve seen, gives us a much more palatable quantified modal logic that nonetheless remains faithful to the intuitions behind the Russellian view of propositions. But propositions are limiting cases of $n$-place relations, and truth-values limiting cases of extensions for relations. Thus, we can generalize the above definition: For any given notion of extension $e$, the extension of an $n$-place relation $R$ is positive at a world $w$ just in case $e(R, w) = T$ if $n = 0$, and $e(R, w) \neq \emptyset$ otherwise. Then on our reconstruction of Adams’ approach as it stands, a relation generally can have a positive extension at a world in which it doesn’t exist only if it is a proposition. But this is an unwarranted restriction. For the notion of truth-at by which positive extensions are assigned to propositions at worlds in which they don’t exist generalizes straightforwardly to a notion of holding-at for $n$-place relations, for all $n$. For instance, consider a world $w$ in which Adams exists but Quine does not. Just as it makes good sense to say that the proposition $[\lambda x a \neq q]$ that Adams is distinct from Quine is true at $w$, it is equally sensible to say that the property $[\lambda x x \neq q]$ of being distinct from Quine holds of Adams at $w$ as well.

This observation requires that we define a general notion of a relation’s extension at worlds that comports with, but extends, the extension functions used to define worlds so as to allow relations generally to have extensions at worlds in which they don’t exist. So let $R_{\pi} = \bigcup_{\pi \in D} R_{\pi}$ and let $R = \bigcup_{w \in W} R_{\pi}$, where $W$ is the class of all possible worlds; ‘$R$, that is, is the class of “all possible” PRPs (and hence, given our definition of worlds, will include “surrogate PRPs” involving surrogate possibilia among their constituents). Then for a given world $w$, we let $\text{ext}_{\pi}^* \supset \text{ext}_{\pi}^*$ be a function that maps the elements of $R$ into appropriate extensions in $D_{\pi}$. In particular, if $R = [\lambda x \varphi]$, then $\text{ext}_{\pi}^*(R) = D_{\pi} \cdot \text{ext}_{\pi}^*((\lambda x \varphi))$. So consider the property $[\lambda x x \neq q]$ and let $w$ be the world just noted in which Adams exists and Quine does not. Then we have $\text{ext}_{\pi}^*((\lambda x x \neq q)) = D_{\pi} \cdot \text{ext}_{\pi}^*([\lambda x x = q]) = D_{\pi} \cdot \{ b \in w | (b, q) \in \text{ext}_{\pi}^*([\lambda x x = y]) \} = D_{\pi} \cdot \{ b \in w | (b, q) \in \text{ext}_{\pi}^*([\lambda x x = y]) \} = D_{\pi} \cdot \emptyset$. So in particular Adams $\in \text{ext}_{\pi}^*([\lambda x x \neq q])$.

Note that “surrogate” PRPs are needed explicitly because the above argument can be generalized in a manner that forces us to consider the extensions of PRPs that would exist if things were different. Consider again a world $w$ in which John Paul II—though, God forbid, not as pope—has a grandson, and consider another world $u$ in which that grandson, and hence the property of being distinct from him, does not exist, but Adams does—the actual world itself will do. Then “looking at” both $w$ and $u$ we can see that, from the perspective of $w$, Adams is no less distinct from John Paul II’s
grandson as from Quine; i.e., figuratively, from our perspective once again we can “project” ourselves into w’s perspective and assign an extension to the property of being distinct from John Paul II’s grandson at a that includes Adams. More generally, we want the following to come out true even if we believe there could have been objects other than those that happen actually to exist:

\[(33) \square \forall x(a \neq x \supset \square((\neg E!x \land E!a) \supset (\lambda y \ x \neq y)a)).\]

Thus, in the possible worlds that constitute our representation of the modal facts, we need representations of the properties there would be if things were different, as indicated by modally and quantificationally embedded terms like ‘(\lambda y \ x \neq y)’ in (33).

This generalization of truth-at significantly affects the way we evaluate atomic propositions with respect to a given possible world. The key difference between truth-in and truth-at as defined above showed up in the evaluation of negated propositions: Unlike P2, A2—that a negated proposition [\neg \psi] is true at w just in case \psi is not true at w—permits negated singular propositions like \neg E!q) to be true with respect to worlds in which they do not exist. Given a notion of extension—viz., \text{ext}_x^w—that allows relations generally to be exemplified with respect to worlds in which they don’t exist, we must now modify A1, which, being defined in terms of \text{ext}_x^w, prevents this. Accordingly all we need to do is replace \text{ext}_x^w in A1 with \text{ext}_x^w:

\[A1^* \quad [Pa_1 \ldots a_n] \text{ is true at w just in case (i) } n = 0 \text{ and } \text{ext}_x^w(P) = \top, \text{ or (ii) } n > 0 \text{ and } (a_1, \ldots, a_n) \in \text{ext}_x^w(P).\]

A2 and A3 then remain as before. If we now think in terms of \text{ext}_x^w rather than \text{ext}_x^w, the generalized serious actualism principle GSA no longer holds; more exactly, we can no longer infer \[E!R\] from the truth of \[R(a_1, \ldots, a_n)\] at a given world. However, since \text{ext}_x^w still maps extensions into \text{D}_w, serious actualism still holds. That is, while we lose (12), \pi \supset E!\pi, we retain SA, \pi \supset E!\pi. Furthermore, we have to relax the restriction on \lambda-conversion in \lambda^R, accordingly to allow for true predications at worlds in which the relation predicated does not exist; so now, it appears, we have instead:

\[\lambda^R: \quad \psi_x \supset (E!y \supset (\lambda x \psi)y), \text{ where for all } i 
\leq n, y_i \text{ is free for } x_i \text{ in } \psi.\]

However, this won’t quite do. The condition in \lambda^R that the denotations of all noncomplex terms occurring in \psi_x exist in the world of evaluation guaranteed that \[\lambda x \psi\] would indicate a legitimate relation in that world. Our observation that the metaphor of perspective enables us to assign extensions to relations in worlds in which they don’t themselves exist caused us to relax this restriction in \lambda^R. However, this relaxation presupposes that the embedded term \[\lambda x \psi\] will always represent, from some perspective, a “possible”

relation; i.e., a relation that exists in some world. There is no longer any guarantee that this will be so. Intuitively, it seems that there could be “incompossible” objects. More exactly, it seems to be possible that there be some object x and possible there be some other object y such that, necessarily, x exists only if y doesn’t and (hence) vice versa; i.e.,

\[(34) \quad \exists x \exists y (E!x \supset \neg E!y).\]

Then there is no world w in which both objects exist, and hence it is not possible that there be a PRP containing both objects as constituents, as the following is a theorem of our logic:

\[(35) \quad \square \forall x \forall y ((E!x \supset \neg E!y) \supset \square \neg E!(\lambda x \psi)),\]

where \psi is any formula containing ‘x’ and ‘y’. If we now count a predication \pi \supset \lambda \psi—a 0-place predication \pi^0, in particular—as false at worlds in which \pi has no denotation, certain instances of \lambda^R yield invalid consequences. Consider, for example,

\[E!x \supset \neg E!y \supset [E!x \supset \neg E!y].\]

\[\square \text{ and } K \text{ yield}\]

\[(36) \quad (E!x \supset \neg E!y) \supset [E!x \supset \neg E!y],\]

from which we derive

\[(37) \quad (E!x \supset \neg E!y) \supset \square [E!x \supset \neg E!y],\]

via universal generalization and \square. But if there could be incompossible objects, i.e., if (34) is true, then (38) is false, as there could not be a proposition of the form [E!x \supset \neg E!y] involving them as constituents. Thinking semantically in terms of our representation of possible worlds, if ‘x’ and ‘y’ take incompossible values when we unpack the quantifiers and modal operators in (38), there is no denotation for [E!x \supset \neg E!y], no world containing a proposition involving those values, and so the antecedent of the conditional would be true and the consequent false.

Now we could take (36)–(38) as a logical proof of the impossibility of incompossible objects, but that seems untoward; logic itself shouldn’t decide such issues. A more seemly solution is to build an additional condition into \lambda^R to account for the possibility of incompossibles. Given Ex, the following will do:

\[\lambda^R: \quad \psi_x \supset (E!y \supset (\lambda E!(\lambda x \psi)) \supset (\lambda x \psi)y), \text{ where for all } i \leq n, \text{ y}_i \text{ is free for } \text{x}_i \text{ in } \psi;\]

i.e., if \psi_x holds, then if all the y_i exist and \lambda x \psi is a possible relation, then \lambda x \psi y. Since \lambda E!(\lambda x \psi) is provable, (36) now follows as a theorem. However, since the proof of \lambda E!(\lambda x \psi) in general—in particular, the proof of \lambda E!(\lambda x \psi) requires \text{Id}, the necessitation of (36) is not provable
in general, and so the above proof of (38) is prevented. Of course, if one
desires to rule out incompossibles explicitly so that, so to say, any $n$
possible objects are composable, then $\land E! [\lambda x \varphi]$ becomes
provable without $\text{Id}$, and the added condition in $\lambda_{\text{RL}}^{*}$ simply becomes superfluous.

What then of $\text{RP}$ in this revised logic? Note that as a limiting case of
$\lambda_{\text{RL}}^{*}$ we have

$$(-E! q \Rightarrow E! [-E! q] \Rightarrow [-E! q]),$$

which by propositional logic, $\Box$, and $\mathbf{K}$ yields

$$\Box E! [-E! q] \Rightarrow \Box [-E! q].$$

But $\Box E! [-E! q]$ is provable from $\text{Id}$, $\text{SA}$, the axiom $E! =$, $\text{Ex}$, $\text{T}$, and the $S5$
axiom $\varphi E! [-E! q] \Rightarrow \Box \varphi E! [-E! q]$. Hence, unlike on our earlier picture,

$$\Box [-E! q] \Rightarrow [-E! q],$$

and thus

$$\Box [-E! q] \Rightarrow \Box [-E! q]$$

are provable, and so the move from (1) to (2) is now valid. This is as it
should be on our generalized perspectival view, since it is no longer required
that a PRP exist in a world to have a positive extension there. However, the premise

$$\Box [-E! q] \Rightarrow \Box [-E! q]$$

in the puzzle now fails, since it requires the discredited (12), specifically, the instance $\Box [-E! q] \Rightarrow E! [-E! q]$; i.e., $[-E! q] \Rightarrow E! [-E! q]$. So the logic remains safe from $\text{RP}$.

GENERALIZING TRUTH-AT II: EXTENSIONS

The move from P1 to A1 then to A1* has provided a broader sense
of truth for propositions with respect to a given possible world that preserves
a much more standard quantified modal logic without abandoning the basic
Russellian metaphysics. The modifications that led to A1 and A1* were
driven by the metaphor of perspective, which provided an intuitive underpinning
to the assignment of positive extensions to PRPs with respect to worlds in
which they don’t exist without violating actualist scruples. Given that metaphor, however, it seems we can go further still. For if we do indeed have something like a perspective on other worlds, can we not, from our perspective in the actual world, consider a property’s extension at a given
world to encompass not just objects in that world, but in other worlds as
well? Consider, once again, a world $w$ in which Quine does not exist. Then,
from our perspective, it seems quite possible to consider Quine to have the
property nonexistence at—though not in, to be sure—$w$. As the example shows, once again the point applies most directly to negative relations. There are, in particular, two ways in which I can be included in the extension of a
negative property $\land x \neg \varphi$ at a world $w$: I can exist in $w$ and be among the
things that have $\land x \varphi$, or I can simply fail to exist in $w$, and hence from the perspective of the actual world can be counted among the things that exemplify $\land x \neg P x$ at $w$.

To incorporate this idea into our apparatus of possible worlds requires only
that we allow $ext^*$ to map $n$-place relations into the $n^+$ Cartesian
product of the entire domain $D = \bigcup_{w \in \mathbb{W}} D_w$ of “possible individuals”.
Propositions continue to be evaluated at worlds in accordance with the
principles A1, A2, and A3. Changes in the logic are minimal. First, since objects
can be in the extensions of relations at worlds in which they don’t exist, $\text{SA}$
now fails. And second, we can drop the condition on ‘$y$’ in $\lambda_{\text{RL}}^{*}$ that the
denotations of all the $i$, $i \leq n$, $y_i$ are free for $x_i$ in $\varphi$.

$\text{RP}$ remains blocked as above, as (12) is still invalid. However, without $\text{SA}$,
it is no longer possible to prove such desirable theorems as (31). This can be remedied by simply adding the relevant special case of $\text{SA}$:

$$\text{SA} =: \tau \equiv \tau \Rightarrow E! \tau,$$

for any noncomplect term $\tau$

to the effect that anything standing in the identity relation with itself exists.

All proofs in the previous system relying on the $\text{Id/SA}$ combination now go through just as before.

Note well that to abandon the axiom $\text{SA}$ is not to abandon serious actualism as a metaphysical principle, any more than abandoning Prior’s $\equiv$
($\equiv \neg \varphi \Rightarrow \Box \neg \varphi$) for $\equiv^*$ is to abandon the metaphysics of Russellian propositions.
Our logical principles are the resultant of two vectors: our metaphysics and the semantic principles we use to evaluate (the propositions expressed by)
by sentences of our logical language. Our underlying Russellian metaphysics is reflected in the definition of a possible world $\langle D_w, ext^* \rangle$ as
insofar as every possible state of affairs must satisfy the principle $\text{Ex}$. $\equiv^*$ reflects both a commitment to the metaphysics and an internalist modal semantics. As is evident in the discussion of Adams’ approach, the move to $\equiv^*$ reflected no change in the basic metaphysics; the definition of worlds remained the same, and, in particular, it remained the case that all and only those propositions exist whose constituents also exist. Rather, it reflected only the switch to a perspectival semantics, a change in the way propositions are evaluated with respect to worlds, as indicated by the switch from the semantic principles $P1$–$P4$ to $A1$–$A3$.

Precisely the same point applies to our abandonment of $\text{SA}$ (and the
move from $\lambda_{\text{RL}}^{*}$ to $\lambda_{\text{RL}}^{*}$). Both reflect only a change in our semantic principles, not in our metaphysics. Given the metaphysics, we are able to derive a broader notion of exemplification in terms of the notion of perspective, on the basis of which we arrive at our new semantic principle $A1^*$.37
Of course, another way to abandon SA is to abandon the metaphysics by allowing the properties and relations in a given world to take values outside that world; i.e., in terms of our apparatus, to allow \( \text{ext}_w \) to take properties and relations to extensions that include objects that don’t exist in \( w \). But why not? We define \( \text{ext}_w \) in such a manner, why not \( \text{ext}_w \)? How do we show ourselves more committed to actualism by restricting the range of \( \text{ext}_w \) and then defining \( \text{ext}_w \) without any such restriction? Why not just allow \( \text{ext}_w \) itself to take values at other worlds from the outset?

The reason for this is that each world structure \( w = (D_w, \text{ext}_w) \) in our framework in itself represents (in general, by means of surrogates) a way things would have been if that world had been actual: All the objects there would have been—the role of \( D_w \)—and the way that those objects would have been configured—the role of \( \text{ext}_w \). An actualist will therefore not include anything in \( D_w \) that is not in the extension of existence \( E! \) (this is a condition on world structures, recall), and a serious actualist will not include anything but such existing things among the extensions of properties and relations. (Indeed, since to be in the extension of a property or relation is to be in some sense, it is hard to see how an actualist could not be a serious actualist.) Once we are given an array of such structures, it provides a fixed, objective representation of the modal facts in terms of which to cash the notion of perspective, and thereby develop the semantic foundations for our alternative modal logics. Again, though, by representing things as they could have been with actually existing objects, we capture the modal facts without any ontological commitment to individuals, propositions, properties, and relations that don’t exist in fact but would have existed had things been otherwise. In this way actualism, serious actualism, and the RusselLian metaphysics of singular properties, relations, and propositions are all preserved alongside a robust quantified modal logic.

APPENDIX: THE LOGICS

In this appendix we assemble the four actualist logics in one place. Let \( E! \) abbreviate \( \{x \exists y (y = x)\} \), and let \( x \) and \( y \) be arbitrary nonrepeating sequences of variables of any length \( n \geq 0 \). Where \( \tau = \tau_1 \ldots \tau_m \) is a nonrepeating sequence of terms, we write \( E! \tau \) to abbreviate \( E! \tau_1 \wedge \ldots \wedge E! \tau_m \). The PRIOREAN SYSTEM Q

The following is Prior’s full system Q of quantified modal logic, formulated in terms of our richer language. Let \( [\Box \varphi] \equiv_{df} \sim \Box \sim \varphi \).

**Taut:** Propositional tautologies.

**PK:** \( [\varphi \supset \psi] \equiv (E! \tau \supset [\Box \varphi \supset [\Box \psi])] \), where \( \tau \) contains all the terms occurring free in \( \varphi \) but not \( \psi \).

**PT:** \( \varphi \supset \Box \varphi \).

**PS:** \( \Box \varphi \supset [\Box \varphi] \).

**QI:** \( [\Box \varphi] \equiv [\Box \varphi] \).

**Qu:** \( \forall x (\varphi \supset \psi) \equiv (\varphi \supset \forall x \psi) \), if \( x \) is not free in \( \varphi \).

**UI:** \( \forall x \varphi \supset \varphi^x \), where \( x \) is free for \( x \) in \( \varphi \).

**GSA:** \( \varphi \equiv E!(\varphi) \), where \( \varphi \) is atomic.

**Ex:** \( E![\lambda \varphi \supset \tau] \equiv E! \tau \), where \( \tau \) contains all and only the noncomplex terms other than the \( \chi_i \) that occur free in \( \varphi \).

**□Ex:** \( [\Box E! \lambda \varphi \supset \tau] \equiv [\Box E! \tau] \), where \( \tau \) is as in **Ex**.

**□E!**s: \( E! \tau \).

**Id:** \( x = x \).

**LL:** \( x = y \supset (\varphi \supset \varphi') \), where \( \varphi \) is atomic, and \( \varphi' \) is just like \( \varphi \) except that \( y \) replaces one or more (free) occurrences of \( x \) in \( \varphi \), and where \( y \) is free for \( x \) in \( \varphi \).

**λ-con:** \( \lambda \varphi = [\lambda x] \lambda \varphi \), where for all \( i \leq n \), \( \chi_i \) is free for \( x_i \) in \( \varphi \).

**PRP:** Fine-grained axioms for PRPs.

**Rules of Inference**

**I:** \( \Gamma \varphi \rightarrow \Gamma [\Box \varphi] \).

**PMP:** \( \Gamma \varphi \rightarrow \Gamma \varphi' \rightarrow \Gamma \varphi' \).

**PGen:** \( \Gamma \varphi \rightarrow \Gamma [\Box \forall x \varphi] \).

THE SYSTEM A1

It is easiest to define this system and the ones below based on Adams’ approach by means of two logics, one a free sublogic of the other. We start first by defining a sublogic of both systems, called **SI**, which consists of the following.

**Taut:** Propositional tautologies.

**K:** \( [\Box (\varphi \supset \psi)] \equiv (\Box \varphi \supset [\Box \psi]) \).

**T:** \( [\Box \varphi] \equiv [\Box \varphi] \).

**S:** \( [\Box \varphi] \equiv [\Box \varphi] \).

**FUI:** \( \forall x \varphi \equiv (E! \tau \supset \varphi^x) \), for any term \( \tau \) that is free for \( x \) in \( \varphi \).

**Qu1:** \( \forall x (\varphi \supset \psi) \equiv (\forall x \varphi \supset \forall x \psi) \).

**Qu2:** \( \varphi \equiv \forall x \varphi \), where \( x \) is not free in \( \varphi \).

**GSA:** \( \varphi \equiv E!(\varphi) \), where \( \varphi \) is atomic.

**Ex:** \( E! [\lambda \varphi \supset \tau] \equiv E! \tau \), where \( \tau \) contains all and only the noncomplex terms other than the \( \chi_i \) that occur free in \( \varphi \).
THE SYSTEM A3
Let S3 be the system that results from adding to S2 the axioms and rules of inference \( \text{Gld}, \neg \text{E}!, \text{GMP}, \text{GGen}, \text{and GNec} \), except with the subscript ‘G1’ replaced by ‘G3’.

By A3 we will mean the system that results by adding to S3 the axioms and rules of inference \( \text{Id}, \text{MP}, \text{Gen}, \text{and Nec} \), except with the subscript ‘G1’ replaced by ‘A3’.

NOTES


3. As illustrated here, I will often use boldface metavariables to indicate sequences of terms of the corresponding type.


6. Cf. Menzel, op. cit., where I argue for the value and propriety of such a language at length.


8. It is an important advantage of dual-role syntax that the intuitive distinction between these two propositions can be expressed syntactically. If a-terms cannot occur in predicate position, then one is tempted to express both the predication and the modalization by means of the same abstract ‘(A0–E4)’, leaving one unable, in particular, to sort out some of the details of the Russellian puzzle.

9. In the usual Lewis/Stalnaker types of semantics for counterfactuals, anyway. But as Chris Swyter reminded me, the equivalence is not intuitive for some counterfactuals with necessarily false antecedents. For example, consider the necessarily false proposition A that arithmetic has a complete recursive axiomatization. On the usual semantics, any counterfactual with a necessarily false antecedent like A is true. But it is intuitively false that if
10. See A. Plantiga, "On Existentialism," Philosophical Studies 44 (1983): 1–20. I should note that the three non-Priorian solutions to RP in this paper also provide replies to the anti-existentialist argument Plantinga constructs on 9–10 of this important article.

11. Recall that $E_{x} E_{y} (Ax \exists y (x = y))$. By $E_{x}$ we have $E_{x} E_{y} (Ax \exists y (x = y))$ (since $x$ is the only noncomplex term occurring in $[Ax \exists y (x = y)]$), and by $\forall x$ we have $[E_{x} E_{y} (Ax \exists y (x = y))]$, and by $K$ it follows that $E_{x} E_{y} (Ax \exists y (x = y))$, and so by the necessitation of $E_{x}$ we have $E_{x}! (Ax \exists y (x = y))$.

12. To see this in the case of (12) (hence (11) as well in the 0-place case), suppose $\pi \tau$. By GSA we have $E_{x} \pi! (x \pi! \tau)$. If $\pi$ is a primitive predicate, then by $E_{x}$ we have $E_{x}! (Ax \exists y (x = y))$, and by $E_{x}$ we have $E_{x}! (Ax \exists y (x = y))$, and by $E_{x}$ we have $E_{x}! (Ax \exists y (x = y))$, and so by the necessitation of $E_{x}$ we have $E_{x}! (Ax \exists y (x = y))$.


14. A more careful account will of course have to qualify matters somewhat: e.g., $D_{x}$ can’t be expected to contain all sets, else it wouldn’t itself be a set. We could take it to be a class, but then the definition of a state of affairs as an ordered pair must be revised (as classes cannot be members of sets in standard set theory). Another option is to define worlds to be maximal states of affairs relative to some property true of all the elements of $D_{x}$; cf. C. Menzel, "The True Modal Logic," Journal of Philosophical Logic 20 (1991): 331–374, esp. 371–2, n. 27. Important as it is, however, we will not have the luxury of pursuing this issue here.

15. Since such surrogate can themselves be elements of possible states of affairs, it might well be necessary in general systematically to map every set to some representational counterpart so as not to confuse a set in its representational role with the set per se. That is, for instance, letting the pure sets be our surrogate possibility, we can then represent sets themselves by some impure but necessarily existing counterpart, say, an ordered pair whose first element is the number 0 (or, if you believe numbers are sets, the identity relation). Such a pair (0, 0) would itself in turn be represented by the pair (0, 0), and so on. Thus, the mapping from surrogates to existing objects mentioned in the definition of states of affairs must also meet the condition that surrogates for sets always be mapped to their correct non-surrogate counterparts. Additional care has to be taken when considering sets of such surrogate worlds for semantic purposes so as to ensure that the same surrogate plays the same role in each world. The details of these considerations are worked out in rather excruciating fashion in C. Menzel, "Actualism, Ontological Commitment, and Possible World Semantics," Synthese 85 (1990): 355–389. Greg Ray has tightened up and formalized these ideas handsomely in "Ontology-Free Modal Semantics," forthcoming in the Journal of Philosophical Logic: A related, more formal approach is found in "The True Modal Logic," esp. 350–2.

16. It is important to note that the term ‘the set of existing objects’ has narrow scope here; that is, it occurs within the scope of the possibility operator. Vulgarly put, then, we are not necessarily talking about an actually existing set here, but about a set of objects that would have existed had things been different.

17. The logic that follows is very close to Prior’s own quantified modal logic for contingent beings, but is much more explicit due to the more expressive language in which it is couched. I reconstruct and criticize Prior’s logic in detail, and provide a more appealing alternative, in "The True Modal Logic."

36. It is perhaps not obvious that this should be so in our fully perspectival semantics. While it seems clear that I don’t stand in, e.g., the acquaintance-of relation with my friend Pete Lundstrom at a world \( w \) in which I don’t exist, and that I do stand in its complement with him at \( w \), is it clear that I don’t stand in the identity relation with myself at \( w \)? Is it clear that I am better characterized as being identical with myself or not at worlds in which I fail to exist? My intuitions side with the latter, but the former is consistent. For its implications on the logic, see note 39.

37. Failure to appreciate this point, I think, has led some actualists to deny serious actualism; see, e.g., Pollock’s arguments in “Plantinga on Possible Worlds,” 126ff, and N. Salmon, “Existence,” 91ff. I’d like to think that all they are really doing is denying \( \mathbf{SA} \) as a logical principle.

38. These axioms ensure, in particular, that PRPs with distinct constituents or distinct logical forms are themselves distinct. Since it is rather tedious to introduce the definitions needed to express these axioms precisely, and since nothing hinges on their precise expression for the purposes of this paper, I will avoid listing them here.

39. To incorporate into \( \mathbf{A3} \) the principle that contingent objects stand in the identity relation with themselves even at worlds in which they don’t exist we replace the axiom \( \mathbf{SA} = \mathbf{In} \) with the axiom schema \( \mathbf{Id} \) (so \( \mathbf{\Box} \mathbf{r} = \mathbf{\tau} \) will now be provable in \( \mathbf{A3} \)) and drop \( \mathbf{NI} \) (as the stronger \( \mathbf{\tau} = \mathbf{\tau} \mathbf{\Box} \mathbf{\tau} = \mathbf{\tau} \) becomes provable from \( \mathbf{Id}, \mathbf{Nec}, \) and \( \mathbf{Ind} \)), replace the axiom \( \forall \mathbf{x} (\mathbf{x} = \mathbf{x}) \) with \( \forall \mathbf{x} \mathbf{E} \mathbf{x} \mathbf{x} \) in \( \mathbf{G2} \), and replace \( \mathbf{Id} \) in \( \mathbf{A3} \) with the axiom schema \( \mathbf{E} \mathbf{\Box} \mathbf{\tau} \), for all noncomplex terms \( \mathbf{\tau} \).

40. My thanks to Ed Zalta for typically incisive comments, and to John Gibbon and my colleagues at Texas A&M for their comments on a presentation based on an early draft of this paper. Thanks also to Chris Hill for numerous improvements stemming from a meticulous reading of the penultimate draft of the paper, and also for his exemplary editorial patience.

PHILOSOPHICAL TOPICS
VOL. 21 NO. 2, SPRING 1993

The Refutation of Nominalism (?)

Gideon Rosen
Princeton University

I. INTRODUCTION

Philosophers disagree (of course) about whether nominalism is true. They disagree about whether it’s plausible. They even disagree about whether it’s worth discussing. And yet beneath this profound disagreement there exists a significant consensus about what kind of problem the problem of nominalism is. According to this consensus, nominalism and its opposite, platonism, are both intelligible, coherent claims about what exists. In an older vocabulary, they are both synthetic ontological hypotheses. This means the dispute cannot be settled a priori. All the reflection in the world will not turn up a contradiction in either proposal. At the same time, however, the dispute cannot be settled by ordinary empirical means either. Both sides agree that causally inert, non-spatiotemporal abstract objects would be radically unobservable if they existed. Numbers and the rest, unlike neutrinos, do not leave the sorts of traces in the sensible world that a scientist can set about trying to find. What saves the dispute from being simply intractable is the recognition that large scale theoretical hypotheses are never settled by mere observation, but rather by considerations of overall theoretical utility, beauty, coherence with settled doctrine, and so on. The open question is therefore whether a systematic world view that posits abstract things over and above the concrete objects of our experience fares better by these familiar criteria than one that doesn’t. And here there is apparently room for dispute.